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Mathematical Modelling of the Process of Vibration Mixing of Minced Sausage

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Abstract. The relevance of the study is determined by the need to optimise the process of mixing minced meat in the production of sausages “Ozdorovchi” to reduce the duration of the technological operation, energy and raw material costs. Therefore, the article is concerned with the substantiation and determination of the amplitude-force parameters of a vibro-mechanical machine for mixing sausage mince ingredients. The leading method for studying this issue is mathematical modelling, which allows for comprehensive consideration of the patterns of changes in the main parameters of the oscillatory process. The article theoretically substantiates the expediency of using vibration impact to provide a rapid and uniform distribution of components in the minced mass. An experimental model of a vibrating machine for mixing minced sausage ingredients has been developed. The calculation scheme of the investigated technical system of the process of vibration mixing of minced sausages “Ozdorovchi” is compiled. The equation of motion of the executive bodies of the system is compiled. The dependences for the equations of motion of the actuators of the vibration mixer are determined. Dependences for the main characteristics of the oscillatory system under study are calculated. Based on the analysis of the graphical representation of the amplitude-force dependences, the working amplitude of oscillations of the mixing tank is substantiated in the range of 2-2.5 mm. It is established that the implementation of the required operating oscillatory mode requires for a given capacity relatively small power consumption in the range of 500-600 W. Optimal parameters for mixing minced meat with vibration intensification of the process, as well as the use of appropriate ingredients, allow for achieving a comprehensive technological effect while minimising energy consumption. Therefore, the materials of the article are of practical value for the meat processing industry in the technology of boiled sausage products with health-improving properties

Keywords: low-frequency oscillations, Lagrange method, oscillatory system, amplitude-frequency characteristics, power characteristics

Introduction

One of the largest components of the diet of a modern person is meat products. According to the World Health Organisation, their share in the total volume of consumption is about a quarter and among those, the largest part is accounted for by sausage products, ranging around 60% [1]. However, their consumption can lead to negative health consequences since minced sausage contains from 2 to 6% table salt. Excessive salt intake leads to deterioration of health, which is manifested in the accumulation of water in the body, stretching of muscle ligaments and deterioration of muscle contractility, inflammation of the kidneys, kidney failure, impaired impulse transmission in the brain, haemorrhage and increased risk of stroke, hyperactivity, and excessive excitability [2]. Sausage products also contain potentially dangerous sodium nitrite for the health of consumers, the ingestion of which can cause the formation of undesirable nitrozoamines, which are carcinogenic compounds [3; 4] therefore, important issues in the development of sausage products with health-improving properties are reducing the level of intake of sodium cation and

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reducing the dosage size in the nitrite ion formulation. Given this, the recipe of sausages "Ozdorovchi" has been improved by reducing the dosage of sodium cation in the minced meat by replacing the usual table salt with sea salt and reducing the dosage of toxic additive nitrite ion in the minced meat by introducing a mixture of antioxidants of natural origin [2; 5]. Changes in the recipe of sausages lead to the improvement of traditional technology and optimisation of the main technological processes due to the introduction of functional additives into their composition; improvement of contact interaction conditions in the loading mass when providing low-frequency vibrations to the executive bodies of the mixer.

One of the most complex and technologically significant processes in sausage production technology is the process of mixing the ingredients of minced sausage. When mixing the components of the minced meat mixture, complex colloidal, physicochemical and biochemical transformations occur under the influence of water and enzyme systems [6]. The course of these processes depends both on the characteristics of the chemical composition and physical and mechanical structure of raw materials, and on the laws of vibro-mechanical mixing. Therefore, to solve pressing issues of the technology of cooked sausage products and impart them with health properties, it is important to study the feasibility of using vibro-mechanical exposure in the mixing process. In the technology of sausages "Ozdorovchi" vibromechanical mixing of ingredients is provided after the initial grinding of meat.

Among the mechanical methods of contact interaction, the use of vibration or low-frequency oscillations makes it possible to transfer the largest energy flux to the system with a small amplitude of displacement of its working bodies during the oscillation period; the ability to influence both significant volumes of products and very limited areas of it; a significant increase in the surfaces of the interaction of technological media up to 100% of the free surface, an increase in the diffusion rate up to 3-3.5 times compared to convection, a decrease in the effective density of the material and a change in other rheological or structural and mechanical properties of the material.

Vibration can be considered a universal form of mechanical effects on processed materials and is widely used during the mixing of technological masses, such as grain, cereals, and meat raw materials, which is one of the most common processes of food production, in particular, primary processing of agricultural raw materials and products [7]. In the "vibration field", the adhesion between the particles of the technological mass can be reduced for bulk masses almost up to 10 times, which leads to the states of pseudo-liquefaction and pseudo-fluidity in the system. In the conditions of elastic and visco-plastic masses, similarly, favourable conditions are created for effective mixing of the mass due to an increase in the area of contact interaction, which reduces the cost of moving the material inside the working 1.5-2 times; due to the reduction of internal friction and mobility of instantaneous equilibrium centres of the particles of the technological load, acceleration of diffusion processes, in particular, salting of meat products, can potentially save energy costs for the operations by 2.5-3 times [9; 10].

Finding the effective regime settings of the investigated process according to the amplitude-power characteristics allows considering both the improvement of contact interaction conditions in the minced meat mass and the reduction of processing time and the corresponding reduction of energy consumption for the process.

In scientific studies, considerable attention is given to the application of vibration in the processes of mixing and the separation of solid bulk heterogeneous systems [11; 12]. According to the results of studies of vibration movement of fine bulk raw materials, the movement of bulk solids in a vessel that performs circular and translational oscillations is examined, the theoretical foundations of layer-by-layer movement of masses on a vibrating surface are developed, recommendations for the process of vibration separation of bulk mixtures are substantiated. In particular, to intensify the process of vibratory sieving of grain, crushed grain feed, the amplitude-frequency characteristics and design parameters of the vibration exciter of the separator of volumetric vibrations are substantiated [9; 10]. The design parameters of the vibratory aspiration separator were developed to improve the quality of separation and clear separation of sunflower seed fractions [13].

Vibration exposure is widely used for the transportation of raw materials and supplies. Thus, the use of vibration conveyors makes it possible to combine the process of material transportation with its technological processing [14].

It is known that due to vibro-mechanical activation of the medium, technological processes are intensified and their productivity increases [15]. The effectiveness of the use of vibration in the drying process has been proved and the operating parameters of vibratory conveyor infrared dryers have been substantiated [16; 17] It has been established that the vibrating wave conveyor infrared dryer is competitive and superior to existing dryers by generalising indicators of two types due to a significant effect on energy consumption and metal consumption [17; 18]. The positive effect of the process of vibro-mechanical activation of hydrolysis of plant material for pectin extraction was investigated [19].

The use of vibration in the meat processing industry can significantly improve traditional and develop new technological processes based on the acceleration of heat and mass transfer processes. The effectiveness of using wave effects on the intensification of the salting process and meat maturation was confirmed [20]. The use of vibration impact improves the conditions of contact interaction and heat and mass transfer, which allows for reducing the processing time by 2-3 times, as well as the conditions of maturation of meat raw materials, and even distribution of the components in the volume and improve the quality of saturation of raw materials with functional ingredients. Optimisation of the technological process of mixing is necessary to ensure such basic technological requirements for the functional properties of minced cooked sausages, as a bound state of moisture and fat during technological processing and in the finished product, the maintenance of a monolithic structure, juiciness and appropriate organoleptic characteristics [5].

According to the results of the analysis, it is possible to draw conclusions about the expediency of vibro-mechanical influence on the process of mixing minced meat ingredients and the optimisation of this technological process using mathematical modelling methods.

Purpose of the study is to substantiate the operating parameters of the process of vibro-mechanical mixing of minced meat ingredients in the technology of "Ozdorovchi" sausages by determining the amplitude and power parameters of the investigated oscillatory system.

The main objectives of the investigation were: to analyse the application of vibro-mechanical exposure in the technology of food production, to develop a system of vibrating machine for mixing the ingredients of minced sausages "Ozdorovchi" and the design scheme of the studied technical system; to draw up the equations of motion of the executive bodies of the system and determine their dependencies; to determine the patterns of change in the main parameters of the oscillatory process and to substantiate the amplitude and power parameters of the mixing tank.

Materials and Methods

Experimental and analytical studies were conducted on the basis of the National University of Life and Environmental Sciences of Ukraine in the laboratory of the Department of Processes and Equipment for Processing of Agricultural Products (agro-industrial complex) during 2020-2021.

To perform the tasks set at the first stage of the experiment, an experimental model of a vibrating machine for mixing minced meat ingredients was developed (Fig.1) [21].

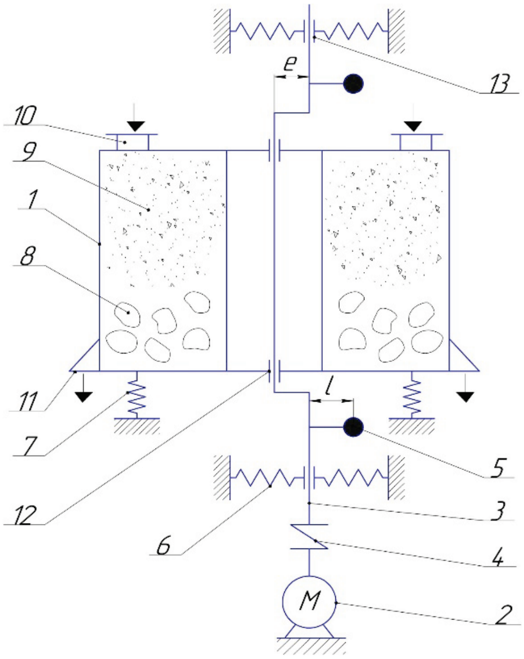

Figure 1. Scheme of a vibrating machine for mixing minced meat ingredients in the production of "Ozdorovchi" sausages:
1 – container, 2 – electric motor, 3 – drive shaft, 4 – flexible coupling;
5 – counterweight, 6 – elastic element of the drive shaft, 7 – elastic element of the container,
8, 9 – minced ingredients, 10 – product supply pipe, 11 – discharge pipe;
12, 13 – support units of the drive shaft; e – eccentricity of the drive shaft,
l – distance from the centre of mass of the counterweight to the axis of the drive shaft

The investigated oscillating system contains a working container 1 (Fig. 1), which rests on vibration supports 7 and bearing assemblies 12 of the drive shaft 3, which is displaced by the value of eccentricity *e* relative to the support elements 13. The latter are equipped with elastic elements 6 to level out parasitic vibrations that can be transmitted from the vibration of container 1. The eccentric shaft receives torque from motor 2 through an elastic coupling 4, which prevents the transmission of vibrations to the driving body. Installation of counterweights 5 on the drive shaft compensates the inertial forces from the moving masses of the technical system under study. Meat products 8 and necessary ingredients 9 enter the working container through the necks 10, and the processed products are removed from the processing area by means of pipes 11.

At the next stage of the experiment, a calculation scheme was developed to determine the amplitude-frequency and power parameters of the studied technical system of the minced meat vibration mixing process (Fig. 2).

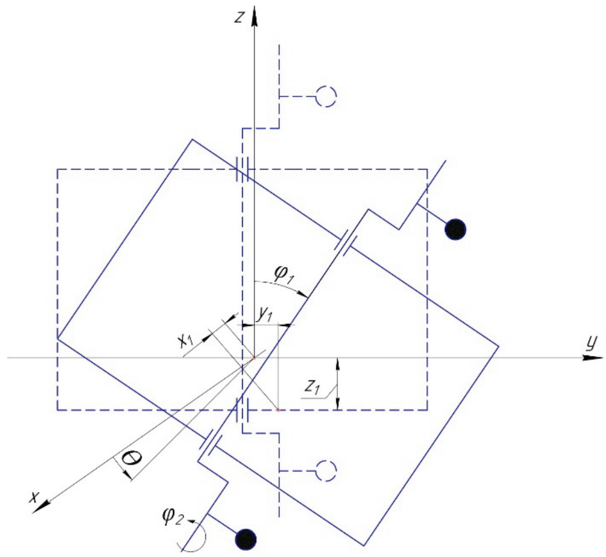

Figure 2. Calculation scheme of the studied technical system of the process of vibration mixing of minced sausages "Ozdorovchi": z_1, in_1, X_1, φ_1, φ_2 – degrees of freedom of working bodies, θ – the angle of rotation of the container relative to the axis X.

The developed oscillatory system is characterised by six degrees of freedom, namely: displacement of the centre of mass relative to the Z coordinate *axes*$_1$, in_1, X_1; by rotating the drive shaft at an angle φ_2; angular displacement of the centre of mass m_1 relative to the OZ axis – φ_1; angular displacement of the centre of mass relative to the Ox – θ *axis*$_1$ (Fig. 2).

Among the moving masses of the system (Fig. 2) the following (1), (2) can be distinguished:

$$m_1 = m_K + \xi_m m_3 \quad (1)$$

$$m_2 = m_B + m_{\Pi}, \quad (2)$$

where m_K – container weight; m_3 – mass of technological loading; ξ_m – coefficient of attached mass of technological loading; m_{Π} – mass of the drive shaft; m_B – mass of counterweights.

Further steps in the implementation of the experiment were drawing up equations of motion of the executive bodies of the system and determining their dependence.This result was achieved using the LaGrange method, for which the equations of motion of the executive bodies of the system were compiled and their mathematical analysis was carried out.

In the mathematical modelling of the developed technical system, the Lagrange method was used, which made it possible to estimate the influence of the kinematic and power parameters of the investigated oscillating system separately for each of the degrees of freedom of the executive bodies of the vibration drive of the machine [23; 24]. The obtained dependencies were processed in the MathCAD mathematical environment.

To complete the tasks set, at the second stage of the experiment, studies were conducted on two samples, the composition of which is shown in Table. 1. The composition of the main ingredients of the control mixture is analogous to the sausages "Lubytelski", LLC "Globino" made according to the standard DSTU 4436:2005 [28], which contains additives recommended for use to give the mixture medicinal properties. The composition of the experimental meat mixture was amended by replacing table salt with sea salt in the traditional sausage recipe and enriching the basic recipe with the *Staphylococcus* bacterial culture, and rosemary and kelp extracts.

***Table 1.** The main ingredients of minced meat*

Ingredient	Control mixture	Experimental mixture
Main raw materials		
Beef	33	30
Semi-fat pork	33	26
Fat pork	34	34
Blood protein	–	1.0
Water to moisturise blood albumin	–	2.0
Cellular tissue (SITRI-Fi 100)	–	0.5
Water for hydration of cell tissues	–	6.5
Total	100	100
Spices and materials		
Table salt sugar	2.2	–
Sea salt with kelp	–	2.1

Table 1, Continued

Ingredient	Control mixture	Experimental mixture
Spices and materials		
Sugar	0.16	0.16
Sodium nitrite	0.0075	0.005
Bacterial preparation (Iprovit LRR)	–	0.05
Rosemary extract	–	0.015
Water	35.0	30.0

The study of the chemical composition was carried out by the following methods: the mass fraction of moisture was determined by drying the product sample to a fixed mass at a temperature of 100-105 °C according to DSTU 8029:2015 [29]; mass fraction of ash by the weight method after mineralisation of the product weight in a muffle furnace at a temperature of 500-600 °C according to DSTU 8718:2017 [30]; mass fraction of lipids by the Soxhlet method, which consists in weighing the fat after extraction with a solvent from the mass of the dry sample in the Soxhlet apparatus, based on determining the change in the sample mass after extraction of fat with a solvent according to DSTU 8718:2017 [30]; mass fraction of protein by determining total nitrogen by the Kjeldahl method. Distillation was carried out on a Velp Scientifica UDK 129 steam distiller (Italy), DSTU 8030:2015 [31].

Determination of the mass fraction of fibre was carried out by removing acid-base-soluble substances from the product according to DSTU 8844:2019 [32]. Determination of the fatty acid content was performed by chromatographic method on the Kupol 55 chromatograph DSTU 7693:2015 [33].

The mineral composition (the content of potassium, calcium, magnesium, phosphorus, manganese, etc.) was determined by atomic emission spectrometry with induction plasma, and the content of heavy metals (lead, cadmium, arsenic, mercury, copper, zinc) was determined by atomic absorption spectrometry in accordance with DSTU EN ISO 11885:2019 [34].

Results and Discussion

According to the LaGrange method, the main equations of motion of its executive bodies were obtained for each of the degrees of freedom of the oscillatory system under study. Previously, the kinetic energy of moving masses and dependences for generalised forces for each of the independent coordinates of the system were determined [23; 24].

To solve these dependences, the kinetic energy of the system is decomposed into two components (3):

$$T = T_1 + T_2; \tag{3}$$

where T_1 – kinetic energy of the container or load, which is defined as (4):

$$T_1 = \frac{1}{2}m_1\dot{x}_1^2 + \frac{1}{2}m_1\dot{y}_1^2 + \frac{1}{2}m_1\dot{z}_1^2 + \frac{1}{2}I_1\dot{\phi}_1^2 + \frac{1}{2}I_1^1\dot{\theta}_1^2; \tag{4}$$

T_2– kinetic energy of mass m_2, which is determined from dependency (5):

$$T_2 = \frac{1}{2}m_2\dot{x}_1^2 + \frac{1}{2}m_2\dot{y}_1^2 + \frac{1}{2}I_2\dot{\phi}_2^2 \tag{5}$$

Then the total kinetic energy of the system is (6):

$$T = \frac{1}{2}m_0(\dot{x}_1^2 + \dot{y}_1^2) + \frac{1}{2}m_1\dot{z}_1^2 + \frac{1}{2}I\dot{\phi}_1^2 + \frac{1}{2}I_1^1\dot{\theta}_1^2; + \frac{1}{2}I_2{\phi_2}^2 \tag{6}$$

where $m_0 = m_1 + m_2$; I_1, I_1^1 –moment of inertia of the container mass relative to the vertical and horizontal axes.

Using the obtained dependences and in accordance with the method of composing the Lagrange equations of the second kind, the partial differentials of the investigated oscillatory system by degrees of freedom (7) are found:

$$\begin{gathered}
\frac{\partial T}{\partial \dot{x}_1} = m\dot{x}_1;\ \frac{d}{dt}\left(\frac{\partial T}{\partial \dot{x}_1}\right) = m\ddot{x}_1;\ \frac{\partial T}{\partial \dot{y}_1} = m\dot{y}_1;\ \frac{d}{dt}\left(\frac{\partial T}{\partial \dot{y}_1}\right) = m\ddot{y}_1; \\
\frac{\partial T}{\partial \dot{z}_1} = m\dot{z}_1;\ \frac{d}{dt}\left(\frac{\partial T}{\partial \dot{z}_1}\right) = m\ddot{z}_1;\ \frac{\partial T}{\partial \dot{\phi}_1} = m\dot{\phi}_1;\ \frac{d}{dt}\left(\frac{\partial T}{\partial \dot{\phi}_1}\right) = m\ddot{\phi}_1; \\
\frac{\partial T}{\partial \dot{\theta}_1} = I_1^1\dot{\theta}_1;\ \frac{d}{dt}\left(\frac{\partial T}{\partial \dot{\theta}_1}\right) = I_1^1\ddot{\theta}_1\ \frac{\partial T}{\partial \dot{\phi}_2} = I_2\dot{\phi}_2;\ \frac{d}{dt}\left(\frac{\partial T}{\partial \dot{\phi}_2}\right) = I_2\phi_2 \\
\frac{\partial T}{\partial x_1} = \frac{\partial T}{\partial y_1} = \frac{\partial T}{\partial z_1} = \frac{\partial T}{\partial \phi_1} = \frac{\partial T}{\partial \theta_1} = \frac{\partial T}{\partial \phi_2} = 0.
\end{gathered} \tag{7}$$

The expression for generalised system forces for all degrees of freedom can be found in the form of the following dependencies (8):

$$\begin{gathered}
Q_{X_1} = F\cos\phi_2 - C_{X_1}x_1;\ Q_{Y_1} = F\sin\phi_2 - C_Y y_1;\ Q_{Z_1} = F\sin\theta - C_Z z_1; \\
Q_{\phi_1} = M_{KP} - M_{O\Pi_1} - C_{\phi_1}\phi_1;\ Q_{Q_1} = F\sin\theta\cdot e - C_Z z_1 R;\ Q_{\phi_2} = M_{KP} - M_{O\Pi 2}
\end{gathered} \tag{8}$$

where $F = m_1 e\varpi_2^2$ – driving force module; M_{KP} – motor shaft torque; $M_{O\Pi}$ – moment of resistance forces to rotation of the container; C_{ϕ_1} – stiffness of elastic elements; R – radius of the container.

The obtained dependences of partial differentials and generalised forces for each of the independent coordinates were substituted for Lagrange equations of the 2nd kind. As a result, a system of differential equations (9) is obtained.

$$\begin{cases}
m_0\ddot{x}_1 = m_1 e\varpi_2^2\cos\phi_2 - C_X x_1 \\
m_0\ddot{y}_1 = m_1 e\varpi_2^2\sin\phi_2 - C_Y y_1 \\
m_0\ddot{z}_1 = m_1 e\varpi_2^2\sin\theta - C_Z z_1 \\
I_1\ddot{\phi}_1 = M_{KP} - M_{O\Pi 1} - C_{\phi 1}\phi_1 \\
I_1^1\ddot{\theta}_1 = m_1 e^2\varpi_2^2\sin\theta - C_Z z_1 R \\
M_{KP2} = M_{O\Pi 2}
\end{cases} \tag{9}$$

At the next stage, the dependences for the equations of motion of the executive bodies of the vibrating mixer are determined. For this purpose, the solution of the equation is found $\ddot{x} + \alpha_x\dot{x} + k_x^2 x = \frac{m_1}{m_0}e\dot{\phi}_2^2\cos\omega_2 t$, as for a second-order linear differential equation with constant coefficients assuming that $\varphi_2 = \omega_2$ – angular velocity of the drive shaft of the vibration exciter; $k_x^2 = \frac{C_x}{m_0} = 464$ Н/м – natural frequency of the system; $\alpha_x = 2\sqrt{k_x^2 - \dot{\phi}_2^2} = 2\sqrt{464 - \omega_2^2}$ – system dissipation factor in the axis direction; $F_m = \frac{m_1}{m_0}e\omega_2^2$ – specific modulus of the driving force.

In this case, the studied equation took the form (10):

$$\ddot{x} + \alpha_x\dot{x} + k_x^2 x = F\cos\omega_2 t \tag{10}$$

The solution of equation (10) can be represented as $x = \bar{x} + x^*$; where $\bar{x} = e^{-0.5\alpha_x t}(C_1 \cos\rho_x t + C_2 \sin\rho_x t)$ – general solution of the equation under study; $x^* = B_1 \sin\omega_2 t + B_2 \cos\omega_2 t$ – its partial solution.

Using the properties of equations with constant coefficients, the following system of equations (11) is compiled and solved:

$$-B_1\omega_2^2 \sin\omega_2 t - B_2\omega_2^2 \cos\omega_2 t + \alpha_x B_1\omega_2 \cos\omega_2 t - \alpha_x B_2\omega_2 \sin\omega_2 t + +k_x^2 B_1 \sin\omega_2 t + k_x^2 B_2 \cos\omega_2 t = F_m \cos\omega_2 t$$

$$\begin{cases} -B_2\omega_2^2 + \alpha_x B_1\omega_2 + B_2 k_x^2 = F_m \\ -B_1\omega_2^2 - \alpha_x B_2\omega_2 + B_1 k_x^2 = 0 \end{cases} \Rightarrow \begin{cases} B_2(k_x^2 - \omega_2^2) + \alpha_x B_1\omega_2 = F_m \\ B_1(k_x^2 - \omega_2^2) = B_2\alpha_x\omega_2 \end{cases} \Rightarrow \begin{cases} B_1 = \frac{F_m\alpha_x\omega_2}{(k_x^2-\omega_2^2)+\alpha_x^2\omega_2^2} \\ B_2 = \frac{F_m(k_x^2-\omega_2^2)}{(k_x^2-\omega_2^2)^2+\alpha_x^2\omega_2^2} \end{cases} \quad (11)$$

Then a separate solution of equation (12) is:

$$x^* = \frac{F_m}{(k_x^2-\omega_2^2)^2+\alpha_x^2\omega_2^2}(\alpha_x\omega_2 \sin\omega_2 t + (k_x^2 - \omega_2^2)\cos\omega_2 t) \quad (12)$$

Given the general solution of equation (10), the desired solution is presented as (13):

$$x = e^{-0.5\alpha_x t}(C_1 \cos\rho_x t + C_2 \sin\rho_x t) + \frac{F_m(\alpha_x\omega_2 \sin\omega_2 t + (k_x^2-\omega_2^2)\cos\omega_2 t)}{(k_x^2-\omega_2^2)^2+\alpha_x^2\omega_2^2} \quad (13)$$

Considering the initial conditions $x_0 = 0$; $\dot{x}_0 = \vartheta_{x_0}$ the differentiation constants (14) and (15) are defined.

$$C_1 = \frac{F_m(\omega_2^2-k_x^2)}{(k_x^2-\omega_2^2)+\alpha_x^2\omega_2^2} \quad (14)$$

$$C_2 = \frac{\vartheta_{x_0}}{\rho_x} - \frac{0.5F_m\alpha_x\rho_x^{-1}(k_x^2+\omega_2^2)}{(k_x^2-\omega_2^2)^2+\alpha_x^2\omega_2^2} \quad (15)$$

$\rho_x = \sqrt{k_x^2 - 0.25\alpha_x^2}$ – frequency of natural vibrations of the system.

As a result, the desired solution took the form (16):

$$x = e^{-0.5\alpha_x t}\left[\frac{F_m(\omega_2^2-k_x^2)}{(k_x^2-\omega_2^2)^2+\alpha_x^2\omega_2^2}\cos\rho_x t + \left(\frac{\vartheta_{x_0}}{\rho_x} - \frac{0.5F_m\alpha_x\rho_x^{-1}(k_x^2+\omega_2^2)}{(k_x^2-\omega_2^2)^2+\alpha_x^2\omega_2^2}\right)\sin\rho_x t + \frac{F_m(\alpha_x\omega_2 \sin\omega_2 t+(k_x^2-\omega_2^2)\cos\omega_2 t)}{(k_x^2-\omega_2^2)^2+\alpha_x^2\omega_2^2}\right. \quad (16)$$

Due to the energy dissipation in the oscillatory system under study, the free oscillations are attenuated, and for the steady-state mode, equation (16) can be represented as follows (17):

$$x = \frac{F_m\alpha_x\omega_2}{(k_x^2-\omega_2^2)^2+\alpha_x^2\omega_2^2}\sin\omega_2 t + \frac{F_m(k_x^2-\omega_2^2)}{(k_x^2-\omega_2^2)^2+\alpha_x^2\omega_2^2}\cos\omega_2 t \quad (17)$$

Using the above methodology, the solution of equation $\ddot{y} + \alpha_y\dot{y} + k_y^2 y = \frac{m_1}{m_2}e\dot{\phi}_2^2 \sin\omega_2 t$ (18) is found:

$$y = e^{-0.5\alpha_y t}\left(\frac{F_m\alpha_y\omega_2 \cos\rho_y t}{(k_y^2-\omega_2^2)^2+\alpha_y^2\omega_2^2} + \left(\frac{\vartheta_{y0}}{\rho_y} + \frac{F_m\omega_2\rho_y^{-1}(0.5\alpha_y^2-k_y^2+\omega_2^2)}{(k_y^2-\omega_2^2)^2+\alpha_y\omega_2^2}\right)\sin\rho_y t\right) + \frac{F_m\left((k_y^2-\omega_2^2)\sin\omega_2 t-\alpha_y\omega_2\cos\omega_2 t\right)}{(k_y^2-\omega_2^2)^2+\alpha_y^2\omega_2^2} \quad (18)$$

where $k_y^2 = \frac{C_y}{m_0} = 1485\,^{\text{H}}/_{\text{M}}$; $\alpha_y = 2\sqrt{1485 - \omega_2^2}$; $\rho_y = \sqrt{k_y^2 - 0.25\alpha_y^2}$

The third component of the movement of the executive organs of the studied machine is found from the system of the following equations (19):

$$\begin{cases} \ddot{z} + \alpha_z\dot{z} + \frac{C_z}{m_0}z = \frac{m_1}{m_0}e\dot{\phi}_2^2 \sin\Theta \\ \ddot{\Theta}_1 = (I_1')^{-1}(m_1 e^2\dot{\phi}_2^2 \sin\Theta - C_z Rz) \end{cases} \quad (19)$$

Assuming that the angular deviation of the container is carried out at a constant angular velocity, i.e. $\omega_1 = \dot{\Theta} = const$, and the received information $\sin\Theta = \frac{C_z R}{em_0}z$ equation is transform (19) to the form

$$\ddot{z} + \alpha_z\dot{z} - k_z^2 z = 0 \quad (20)$$

where

$k_z^2 = \frac{C_z}{m_0}\left(\frac{R}{e} - 1\right) = 82665\,^{\text{H}}/_{\text{M}}$; $\alpha_z = 2\sqrt{k_z^2 - \dot{\phi}_2^2} = 2\sqrt{82665 - \omega_2^2}$

The solution of equation (20) is found in the form (21):

$$z = e^{-0.5\alpha_z t}(C_5 \cos\rho_z t + C_6 \sin\rho_z t), \quad (21)$$

where $\rho_z = \sqrt{k_z^2 - 0.25\alpha_z^2}$

Considering the initial conditions $z_0 = 0$; $\dot{z}_0 = \vartheta_{z_0}$ constant differentiations are defined $C_5 = 0$; $C_6 = \frac{\vartheta_{z_0}}{\rho_z}$

As a result (22):

$$z = e^{-0.5\alpha_z t}\vartheta_{z_0}\rho_z^{-1} \sin\rho_z t \quad (22)$$

The next step was to calculate the dependences for the main characteristics of the investigated oscillatory system.

Using replacement (23):

$$\frac{F_m\alpha_x\omega_2}{(k_x^2-\omega_2^2)^2+\alpha_x^2\omega_2^2} = A_x \cos\phi_x$$
$$\frac{F_m(k_x^2-\omega_2^2)}{(k_x^2-\omega_2^2)^2+\alpha_x^2\omega_2^2} = A_x \sin\phi_x \quad (23)$$

The equation (21) is transformed as (24):

$$x = A_x(\cos\phi_x \sin\omega_2 t + \sin\phi_x \cos\omega_2 t) = A_x \sin(\omega_2 t + \phi_x) \quad (24)$$

where $\phi_x = arctg\left(\frac{k_x^2-\omega_2^2}{\alpha_x\omega_2}\right)$

After certain transformations, equation (24) presents the component of the oscillation amplitude A_x as (25):

$$y = \frac{F_m(k_y^2-\omega_2^2)\sin\omega_2 t}{(k_y^2-\omega_2^2)^2+\alpha_y^2\omega_2^2} - \frac{F_m\alpha_y\omega_2 \cos\omega_2 t}{(k_y^2-\omega_2^2)^2+\alpha_y^2\omega_2^2} \quad (25)$$

Using replacement (26):

$$\frac{F_m(k_y^2-\omega_2^2)}{(k_y^2-\omega_2^2)^2+\alpha_y^2\omega_2^2} = A_y \cos\phi_y$$
$$\frac{F_m\alpha_y\omega_2}{(k_y^2-\omega_2^2)^2+\alpha_y^2\omega_2^2} = A_y \sin\phi_y \quad (26)$$

The equation (25) is presented as (27):

$$y = A_y \cos\phi_y \sin\omega_2 t - A_y \sin\phi_y \cos\omega_2 t = A_y \sin(\omega_2 t - \phi_y) \qquad (27)$$

where $\phi_y = arctg(\frac{\alpha_y \omega_2}{k_y^2 - \omega_2^2})$

Taking into account the expressions (25), the component of the A_y oscillation amplitude is obtained as (28):

$$A_y = \frac{F_m \sqrt{\alpha_y^2 \omega_2^2 + (k_y^2 - \omega_2^2)^2}}{(k_y^2 - \omega_2^2)^2 + \alpha_y^2 \omega_2^2} = \frac{F_m}{\sqrt{\alpha_y^2 \omega_2^2 + (k_y^2 - \omega_2^2)^2}} \qquad (28)$$

The absolute amplitude of vibrations is $A = \sqrt{A_x^2 + A_y^2 + A_z^2}$; for a steady-state mode, vibrations in the vertical direction are attenuated, i.e., $A_z = 0$; which, given the derived dependencies for its components, is (29):

$$A = \frac{m_1}{m_0} e\omega_2^2 \sqrt{\frac{1}{(k_x^2 - \omega_2^2)^2 + \alpha_x^2 \omega_2^2} + \frac{1}{(k_y^2 - \omega_2^2)^2 + \alpha_y^2 \omega_2^2}} \qquad (29)$$

Using dependence (29), the amplitude-frequency response is constructed (Fig. 3).

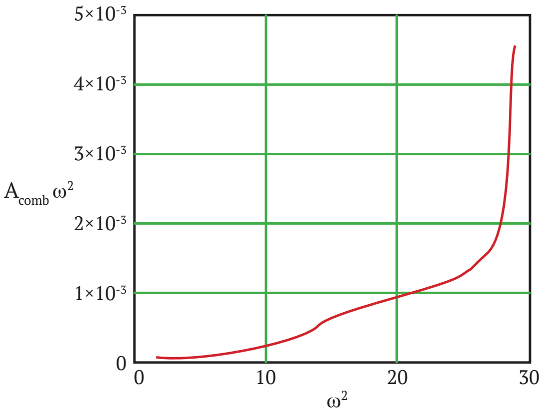

Figure 3. Dependence of the total amplitude of vibrations A_{comb} (m) a vibration mixer with mechanical combined vibration excitation of spatial vibrations from the square of the frequency of forced vibrations ω^2 (rad/s^2)

Expression for the power of the driving force N_F is presented it as a product $N_F = F\vartheta$, where $\vartheta = \sqrt{\vartheta_x^2 + \vartheta_y^2 + \vartheta_z^2} = \sqrt{\dot{x}^2 + \dot{y}^2 + \dot{z}^2}$ – expression for the vibration velocity. The modulus of the forcing force is found as $F = m_1 e\varpi_2^2$.

Considering (30), (31), (32):

$$\dot{x} = \frac{F_m \omega_2}{(k_x^2 - \omega_2^2)^2 + \alpha_x^2 \omega_2^2}(\omega_2 \alpha_x \cos\omega_2 t - (k_x^2 - \omega_2^2)\sin\omega_2 t) \qquad (30)$$

$$\dot{y} = \frac{F_m \omega_2}{(k_y^2 - \omega_2^2)^2 + \alpha_y^2 \omega_2^2}\left((k_y^2 - \omega_2^2)\cos\omega_2 t + \omega_2 \alpha_y \sin\omega_2 t\right) \qquad (31)$$

$$\dot{z} = 0 \qquad (32)$$

The desired expression takes the form in Formula (33), which allowed it to be represented in Figure 4.

$$N_F = m_0^{-1} m_1^2 e^2 \omega_2^5 \sqrt{\left[\frac{\omega_2 \alpha_x \cos\omega_2 t - (k_x^2 - \omega_2^2)\sin\omega_2 t}{(k_x^2 - \omega_2^2)^2 + \alpha_x^2 \omega_2^2}\right]^2 + \left[\frac{(k_y^2 - \omega_2^2)\cos\omega_2 t + \omega_2 \alpha_y \sin\omega_2 t}{(k_y^2 - \omega_2^2)^2 + \alpha_y^2 \omega_2^2}\right]^2} \qquad (33)$$

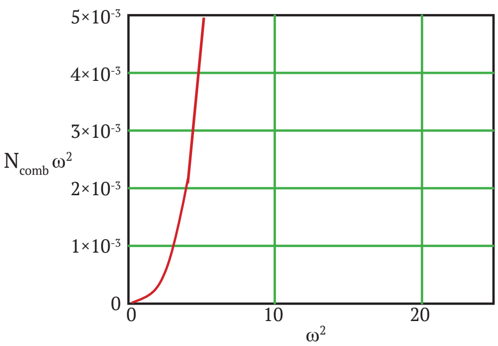

Figure 4. Dependence of the total power consumption N_{comb} (kW) of a vibration mixer with mechanical combined vibration excitation of spatial oscillations on the square of the frequency of forced oscillations ω^2 (rad/s^2)

The total inertial forces from unbalanced masses of the oscillatory system, which characterise the reliability of the support units, are found from dependence (34):

$$F_{unb} = F_{com} = m_1 \cdot a_в;\ a_{in} = A \times \omega^2 \qquad (34)$$

Given the features of the elastic system of the drive mechanism of the developed machine, the presented vibration exciter can be considered as a mechanical combined one, since it combines elements of kinematic forced and unbalanced vibration exciters, respectively, the presence of eccentricity of the drive shaft and an elastic system for levelling parasitic vibrations [9; 23; 24].

Based on the obtained dependence, a graphical representation of this force characteristic is constructed in Figure 5.

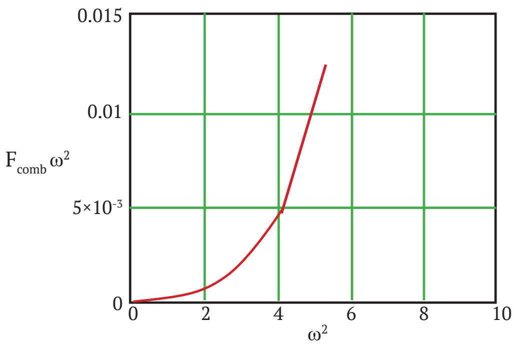

Figure 5. Dependence of the total inertial forces on unbalanced masses F_{comb} (kN) of a vibrating mixer with mechanical combined vibration excitation of spatial oscillations on the square of the frequency of forced oscillations ω^2 (rad/s^2)

The analysis of the presented graphical dependences indicates that the frequency ω^2 exceeds the value of 28 rad/s^2 before the asymptotic increase in the amplitude of oscillations, which can lead to failure of the executive elements of the vibration drive due to the loss of the dynamic equilibrium state by the system, which is observed in systems with power vibration excitation [25]. Therefore, the working amplitude of oscillations of the mixing vessel was chosen in the range of 2-2.5 mm.

The implementation of the above operating oscillatory mode requires for a given container a relatively small power consumption in the range of 500-600 W relative to designs with an unbalanced vibrating exciter, which is satisfactory for the effective processing of minced sausage mass [26; 27].

Conclusions

Based on the analysis of literature sources, the expediency of application of vibration exposure to the process of mixing minced meat in the technology of sausages "Ozdorovchi" is theoretically substantiated.

To improve the technological process of mixing sausage minced meat, a system of vibrating machine for mixing ingredients and a design diagram of the investigated technical system of the process of vibration mixing of minced sausages "Ozdorovchi" were developed.

With the help of mathematical modelling, using the Lagrange method, the equations of motion of the executive bodies of the system are compiled and their dependencies are determined.

Graphical representation of the amplitude-force dependences allowed to substantiate the working amplitude of oscillations of the mixing vessel in the range of 2-2.5 mm and to establish the necessary power consumption in the range of 500-600 W.

Experimental initial data and theoretically substantiated operating parameters of mixing sausage minced meat with vibration intensification of the process, and the application of appropriate recipe ingredients provides for the achievement of a comprehensive technological effect, while minimising energy consumption.

Further research will be focused on the development of the hardware and technological scheme for the production of sausages "Ozdorovchi" using improved technology, considering the determined optimal parameters of vibro-mechanical mixing of minced meat.

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Математичне моделювання процесу вібраційного перемішування сосисочного фаршу

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Анотація. Актуальність дослідження зумовлена необхідністю оптимізації процесу перемішування фаршу в технології сосисок «Оздоровчі» для скорочення тривалості технологічної операції, зменшення енергетичних та сировинних витрат. У зв'язку з цим стаття спрямована на обґрунтування та визначення амплітудно-силових параметрів вібромеханічної машини для перемішування інгредієнтів сосисочного фаршу. Провідним методом до дослідження цієї проблеми є метод математичного моделювання, що дозволяє комплексно розглянути закономірності зміни основних параметрів коливального процесу. В статті теоретично обґрунтовано доцільність використання вібраційного впливу для забезпечення швидкого та рівномірного розподілення компонентів у фаршевій масі. Розроблено дослідну модель вібраційної машини для перемішування інгредієнтів сосисочного фаршу. Складено розрахункову схему досліджуваної технічної системи процесу вібраційного перемішування фаршу сосисок «Оздоровчі». Складено рівняння руху виконавчих органів системи. Визначено залежності для рівнянь руху виконавчих органів віброзмішувача. Розраховано залежності для основних характеристик досліджуваної коливальної системи. На основі аналізу графічного представлення амплітудно-силових залежностей, обґрунтовано робочу амплітуду коливань ємкості для перемішування у межах 2–2,5 мм. Встановлено, що реалізація необхідного робочого коливального режиму вимагає для даної ємкості порівняно невеликих витрат потужності у межах 500–600 Вт. Оптимальні параметри перемішування фаршу з вібраційною інтенсифікацією процесу, а також застосування відповідних інгредієнтів дозволяє досягти комплексний технологічний ефект при мінімізації витрати енергії. Тому, матеріали статті становлять практичну цінність для м'ясопереробної галузі в технології варених ковбасних виробів з оздоровчими властивостями

Ключові слова: низькочастотні коливання, метод Лагранжа, коливальна система, амплітудно-частотні характеристики, силові характеристики